IPM component 3 week 9 – On introductions

What is an introduction?

Here we look at how to write an introduction to a paper. The information in these notes also applies to your extended essay.

The broadest aim of an introduction is to provide background information (this can include literature review) and describe what is to come in the rest of the paper (this can include thesis statement). Then, by looking at as many examples as possible we come to see that the content of an introduction can address several specific themes, each one of which is discussed in more detail in the rest of the essay/paper. Some example of these themes are

1. Giving general background information to the topic;
2. Establish the importance of a topic;
3. Highlighting a problem or controversy
4. Highlighting gaps or inadequacies in previous research;
5. Stating the aim, purpose or focus of the essay; giving a thesis statement;
6. Stating the significance and/or limitation of your work;
7. Outlining the structure and content of your essay.

Our purpose here is to find out what kind of language is used in an introduction. To that end we can ask the question,

What makes a text a/an
{
"general background information"
"establishing the importance of a topic"
"highlighting a problem or controversy"
"highlighting gaps or inadequacies in previous research"
"stating the aim, purpose or focus of the essay"
"stating the significance and/or limitation of your work"
"outlining the srtucture and content of your essay"
} type of text?
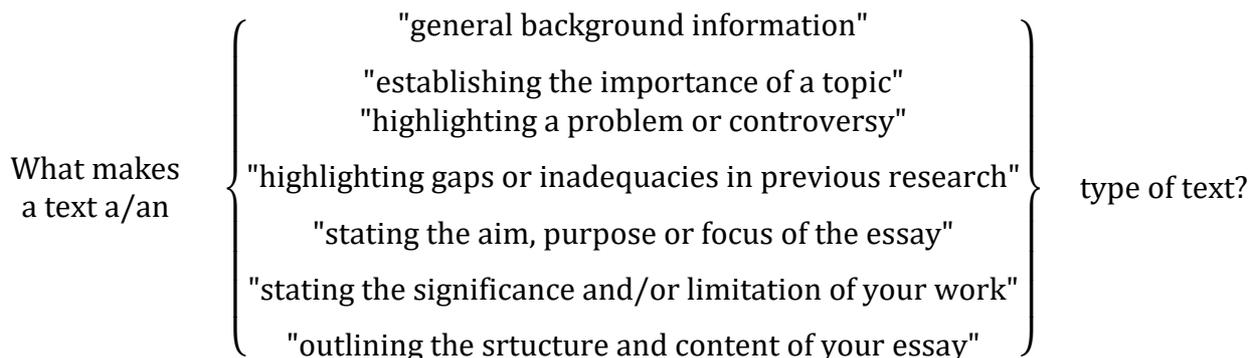
We now look at some examples.

The following text is taken from chapter title: "Mathematics for Big Data", Alessandro Di Bucchianico et al., in *The best writing on mathematics*, Mircea Pitici (Editor), Princeton university press (2019).

1	"The role of mathematics [in big data analytics] is easy to overlook and not fully
2	recognized because technological advances are much more visible than mathematical
3	advances, even though the latter often have more effect. […]
4	In this essay, we present several explicit real–life examples of the mathematics behind
5	big data, highlighting the role and importance of specific areas of mathematics in these
6	contexts. We show a wide variety of examples: search engines, virtual prototyping in
7	manufacturing, data assimilation, web data analytics, health care, recommendation
8	systems, genomics and other *omics* sciences, and precision farming."

Analysing the text

- The first sentence over lines 1-3 acts as topic sentence for the paragraph. Particularly, it seems to suggest that mathematics is underappreciated as a tool in big data analytics. This would suggest that the rest of the paragraph will address this in some way.

- The second sentence of lines 4-6 explicitly refer to the importance of mathematics in big data analytics.

- The third sentence of lines 6-8 then gives examples of where mathematics is used in big data analytics. As such, we might expect the rest of the paper to answer questions such as:

  1. What types of mathematics are used in each of the examples mentioned?

  2. Why is it important to recognise the importance of mathematics in big data analytics?

  3. How does know the answer to the question 1. help professionals in the big data analytics discipline? What will they be able to do with knowledge gained from the answer to 1.?

It is for the authors to decide which questions to raise and answer in their paper. It may be the case that items 2. and 3. are not addressed at all, but such questions seem sensible based on their introductory comments.

1   "[...] I think the arguments given are often rather dubious. For example, teachers often
2   say that the mean is 'unduly' influenced by outliers, but is this right? Surely outliers
3   influence the mean to a precisely proportionate degree? If the outlier is a mistake, and
4   we don't want it to influence our conclusions, we should remove it from our data. If we
5   leave it in, because it is real, then we should expect and want it to have its due
6   influence on our results. An outlier may be the most important piece of data."

Analysing the text
- The first sentence on line 1 suggests a possible problem;
- The second and third sentences over lines 2-3 illustrate this problem by asking questions about the usual way of thinking about the effect of outliers on the mean;
- So, for both the first and the second sentence we would expect either a general answer in this paragraph, with a detailed answer to come later, or a reference to a detailed answer to come later (such as "We will address this in section ...");
- In the fourth and fifth sentence (over lines 3-5) we indeed see answers to the two questions asked. These answers can be considered as generalistic. To see why this is so, questions requiring much more detail can be asked:
    o Under what conditions would we remove or keep the outlier?
    o Can specific mathematical criteria be developed in order to determine whether or not to keep/remove an outlier? Or is such a decision down to the experience of the researcher?

Notice that in order to ask questions of such specificity you have to know the subject!

The following text comes from "Student interpretations of the terms in first order ordinary differential equations in modelling contexts", David R. Rowland & Zlatko Jovanoski, *International Journal of Mathematical Education in Science and Technology*, 35:4 (2004), 503-516.

1	Reform in the teaching of differential equations at the tertiary level has been driven in
2	part by the same sorts of disappointing observations on student learning outcomes
3	that have driven calculus reform [1, 2]. In addition, the ready availability of
4	programmable graphics calculators and symbolic algebra packages such as
5	Mathematica and Maple have raised the question of what it is important for students to
6	be able to do themselves and what can be left for the technology to calculate.
7	Furthermore, these packages have opened up opportunities for more sophisticated
8	analyses and investigations of ordinary differential equations (ODEs) than was
9	previously possible. Consequently, as Boyce [2] describes, in reform efforts there has
10	been a move away from 'mere manipulative skills' teaching, to a greater emphasis on
11	conceptual understanding, exploration and higher-level problem solving.

(from "Student interpretations of the terms in first order ordinary differential equations in modelling contexts", David R. Rowland & Zlatko Jovanoski, *International Journal of Mathematical Education in Science and Technology*, 35:4 (2004), 503-516.

Analysing the text

- The first sentence over lines 1-3 acts as a topic sentence referring to the gaps in research.

- The second sentence of lines 3-6 raise another gap in research this time relating to the importance of the use of software;

- The third sentences of lines 7-9 contrast the idea of the second sentence by highlighting the advantages in the use of such software;

- The fourth sentence of lines 9-11 continues the reference to the advantages of the previous sentence by identifying an underlying principle of teaching.

- As a result of all of the above many questions could be asked, such as:

  o What difficulties do students have in their learning of differential equations?

  o When is it ok to leave students to do things for themselves (presumably without instruction?), and when is it ok to use technology to do the calculations for the student?

Example 4: Stating the purpose, aim or focus of the essay

The following text comes from "Approaching Cauchy's Theorem", Stephan R. Garcia, William T. Ross, 2017, *Math and Computer Science Faculty Publications.* 219, https://scholarship.richmond.edu/mathcs-faculty-publications/219

1	We hope to initiate a discussion about various methods for introducing Cauchy's
2	Theorem. Although Cauchy's Theorem is the fundamental theorem upon which
3	complex analysis is based, there is no "standard approach." The appropriate choice
4	depends upon the prerequisites for the course and the level of rigor intended. Common
5	methods include homotopy theory, Green's Theorem, or Goursat's lemma, each of
6	which has its positives and negatives.[…]
7	One side of the debate argues that technical details are what makes mathematics so
8	beautiful, concise, and complete. It all fits together so beautifully. […]
9	Another side of the debate argues that the majority of our students do not go on to
10	graduate school in pure mathematics. Focusing on the technical details is not needed
11	nor appreciated by the majority of our students. Instead, teachers should emphasize
12	the general ideas and the interconnections of these ideas.[…]
13	In this paper, we focus on Cauchy's Theorem, and look at both sides of this struggle.
14	Since the reader is presumably a professional mathematician with experience in
15	complex variables, we will assume a working knowledge of the field. We do not
16	attempt to work through the details of the proofs and techniques discussed below. We
17	focus instead on several standard approaches to Cauchy's Theorem, and weigh the pros
18	and cons of each [from a pedagogical perspective].

Analysing the text

This type of writing is much easier to read since it should simply list the purpose or focus of the essay. In this case some background reasoning has been provided to justify why the authors will focus on what they choose to focus on.

- The first paragraph over lines 1-6 addresses the many ways in which Cauchy's theorem in taught;

- The second paragraph of lines 7-8 addresses one perspective on how Cauchy's theorem should be presented;

- The third paragraph of lines 9-12 argues against the first perspective, and addresses a second perspective on how Cauchy's theorem should be presented.

- As a result of the background information given in the first three paragraphs (which can be seen as justifying the aim/focus of the paper) the fourth paragraph of lines 13-18 states the aim or focus of the paper. In this sense, it is implied that the paper will highlights various options as to which is the best approach to adopt when teaching Cauchy's theorem.

(note that the text can also be interpreted as highlighting issues or limitation with respect to the teaching and learning of Cauchy's theorem).

The key phrasing is shown as the first and last sentence of the fourth paragraph. Other phrasing which refers to "Stating the purpose, aim or focus of the essay" include:

- "The main objective of this work is to investigate methods for improving …"

- "We are specifically interested in the aspects of … as they relate to …"

- "In this paper we investigate … with specific reference to …"

- "In terms of … this work concentrates on …"

- "In order to address the questions outlined above, we report here …"

Example 5: On the Significance and limitations of your work

The following text comes from

https://www.emathzone.com/tutorials/basic-statistics/limitations-of-statistics.html

1   "The important limitations of statistics are:

2   (1) Statistics laws are true on average. Statistics are aggregates of facts, so a single

3   observation is not a statistic. Statistics deal with groups and aggregates only.

4   (2) Statistical methods are best applicable to quantitative data.

5   (3) Statistics cannot be applied to heterogeneous data.

6   (4) If sufficient care is not exercised in collecting, analyzing and interpreting the data,

7   statistical results might be misleading.

8   (5) Only a person who has an expert knowledge of statistics can handle statistical data

9   efficiently.

10   (6) Some errors are possible in statistical decisions. In particular, inferential statistics

11   involves certain errors. We do not know whether an error has been committed or not."

Analysing the text

This type of writing is much easier to read since it should simply list the significance and limitations of the work. As such, the language and language structure used is fairly standard, two examples of which are illustrated below.

$$\text{The main} \begin{Bmatrix} \text{limitation} \\ \text{effects} \\ \text{constraints} \\ \text{drawback} \\ \text{flaw} \\ \ldots \end{Bmatrix} \text{of X is/are} \qquad \ldots$$

$$\begin{matrix} \text{We know / do} \\ \text{not know} \end{matrix} \begin{Bmatrix} \text{the effect} \\ \text{the consequence} \\ \text{how precise} \\ \text{the response rate} \\ \text{the side-effects} \\ \ldots \end{Bmatrix} \text{of / this} \qquad \ldots$$

1   "This paper draws attention to certain principles governing the application of
2   mathematics to engineering and the physical sciences. Broadly speaking, these
3   applications require

4   *(a)* producing mathematical descriptions of physical phenomena *(modelling),*
5   and
6   *(b)* drawing conclusions about those and related phenomena with the help of
7   such models *(simulation).*

8   Not surprisingly considering its stated goals, this paper is concerned with both
9   modelling and simulation. [...]

10   In the last resort, however, all modelling and simulation work depends upon the
11   expertise and good sense of the modeller; any systematic theory should be aimed
12   at supplementing rather than replacing the modeller's craft. This being said, the
13   reader must be warned that far less ambitious goals are aimed at in most of the
14   paper. For instance, sections 4 and 8 deal with setting up and solving differential
15   equations, respectively. The choice of modelling methodology adopted in section
16   4 owes a great deal to treatments now classical in the chemical engineering
17   literature, such as [7] and [8]. As to section 8, it is a brief review of numerical
18   methods for differential equations, ordinary and partial.[...]

19   The differential equations are ordinary when time is continuous and space is
20   discrete; in all remaining cases there result partial differential equations. To keep
21   matters within bounds, the main ideas developed in both sections 3 and 7 are
22   illustrated in terms of the differential equation models developed in the
23   elementary examples of section 2. Section 7 is concerned with model validation
24   criteria, and relies heavily upon concepts-describing empirical work. It is seen
25   therein that confrontation with experiment may be critical, and require a careful
26   re-examination of the fundamental assumptions underlying the given model."

The analysis of the text is left as an exercise.

Some comments

<u>*Comment 1*</u>

Note the difference between a topic sentence (studied previously) and an introduction:

- the topic sentence is the introduction to a paragraph, whilst the introduction is the introduction to the essay/paper as a whole. Both are forms of introduction (they have the same purpose) but with different content:
  - topic sentences only address one theme, namely that which the paragraph will elaborate on;
  - introductions to papers/essays address multiple themes (see p1).

- the detail of the topic sentence is described in the paragraph for that topic sentence, whereas the detail of the introduction section is described in the rest of the essay/paper.

- a question asked in a topic sentence is usually answered in the paragraph of that topic sentence;

- questions asked in the introduction section of an essay/paper are usually not answered in the introduction but later on in the essay/paper, and in various different sections. If any questions are answered in the introduction, they will usually be answered in a general or vague manner.

So both the topic sentence and the introduction to the essay act as introductions, but with different degrees of generality or specificity, and also at different levels of text (i.e. introductions are written for the paper as a whole whilst topic sentences are written for individual paragraphs).

<u>*Comment 2*</u>

Note that the introduction section of a paper or essay also has its own introductory part. This "introduction to the introduction" usually outlines some general information to justify the body of the introduction.

For example, in the paper "Approaching Cauchy's Theorem" by Stephan Ramon Garcia & William T. Ross (*PRIMUS*, 27:8-9, (2017), 758-765) we have the following:

> "For those of us who regularly teach an undergraduate course in complex variables, there are always the looming concerns about the appropriate level of technical detail and rigor to include. [...] One side of the debate argues that

technical details are what makes mathematics so beautiful, concise, and complete. […] Another side of the debate argues that the majority of our students do not go on to graduate school in pure mathematics. Focusing on the technical details is not needed, […] Since the two of us frequently teach undergraduate complex analysis, we often struggle with "detail" versus "not enough detail," with "plodding proofs" versus "deus ex machina proofs. […] In this paper, we focus on Cauchy's Theorem, and look at both sides of this struggle."

This last sentence falls under the category of "Stating the purpose, aim or focus of the essay", but the authors introduce their aim by preceding it with some background information about the general state of affairs relating to the difficulties in teaching complex analysis course. The same can be seen in the text of example 6. above. It is only in the middle of the second paragraph, and then in the third paragraph, that we see the author describing the structure and content of his paper.

As another example, consider the text below adapted from "Inferential Statistics as Descriptive Statistics: There Is No Replication Crisis if We Don't Expect Replication" by V. Amrhein, D. Trafimow & S. Greenland (*The American Statistician*, 73:sup1, 262-270)

The "crisis of unreplicable research" is not only about alleged replication failures. It is also about perceived nonreplication of scientific results being interpreted as a sign of bad science (Baker 2016). Yes, there is an epidemic of misinterpretation of statistics and what amounts to scientific misconduct, even though it is common practice (such as selectively reporting studies that "worked" or that were "significant"; Martinson, Anderson, and de Vries 2005; John, Loewenstein, and Prelec 2012). But all results are uncertain and highly variable, even those from the most rigorous studies. […]

In the following, we argue that the crisis of unreplicable research is mainly a crisis of overconfidence in statistical results. We recommend that we should use, communicate, and teach inferential statistical methods as describing logical relations between assumptions and data (as detailed in the Appendix), rather than as providing generalizable inferences about universal populations."

Here the last paragraph falls under the category of "Stating the purpose, aim or focus of the essay" and also "Highlighting a problem or controversy". And, in order to be able to state the purpose of their essay, the authors have had use part of their introduction to give some introductory background relating to the problem in the use of inferential statistics.

Ultimately, from all of the examples we have seen we might say that

Introduction = A piece of text focusing on the general aspects of a paper
whose aim is to orient the reader's attention towards the content of the paper,
all this achieved with suitable language and phrasing,
and to a suitable degree of generality.

Note: It is important to note that you do not need to, nor can you, use all of the six themes above in your introduction. The above themes are presented only for you to see the possibilities you have in being able to frame an introduction in the style you want. It is for you to decide how you wish to introduce the aims and reasons for you wanting to present the thesis of your essay.
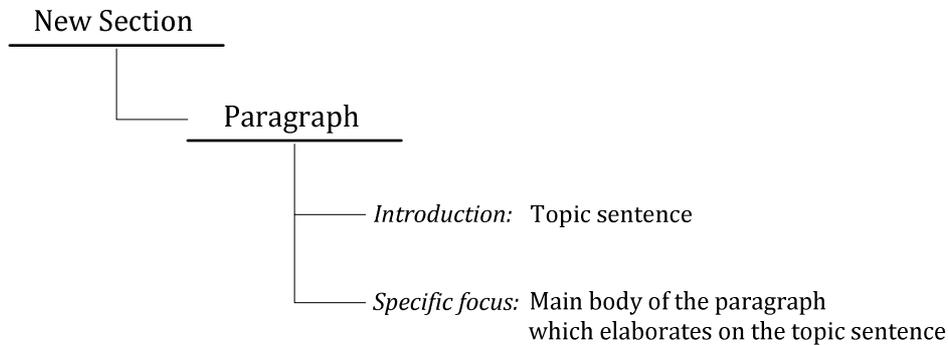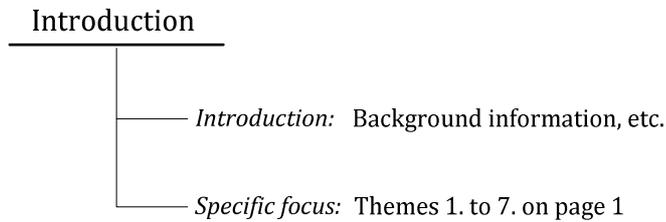
Levels of introduction

Recall that each new paragraph of a text generally has an introduction sentence, i.e. a topic sentence. This topic sentence may become more and more specific as you write your second, third, fourth paragraph since you are now elaborating on the details of the topic of the section.

And, we have seen that the introduction to an essay or paper usually has an introductory part to it. So, it appears that the idea of "introduction" occurs throughout a paper or essay, as illustrated in the diagram below.

The same can be said about the headings of each section of an essay or paper. Each heading can be seen as an introduction to the core theme of a section. For example

- *Title*: We use this word as a "topic" word to orient the reader towards the key idea of the essay or paper.

- *Introduction*: We use this word as a "topic" word to orient the reader towards the fact that we will be giving background information about the topic as well as addressing one of the themes above;

Introduction

    Introduction: Background information, etc.

    Specific focus: Themes 1. to 7. on page 1

New Section

    Paragraph

    Introduction: Topic sentence

    Specific focus: Main body of the paragraph
    which elaborates on the topic sentence

Levels of introduction

- *Methodology*: We use this word as a "topic" word to orient the reader towards the methods that will be used to collect information, as well as the way in which these methods are used systematically;

- *Results and Analysis*: We use these words as "topic" words to orient the reader towards the fact that we will be presenting the results obtained from implementing our methodology, and the analyses of these results;

- *Discussion*: We use this word as a "topic" word to orient the reader towards the fact that we will be discussing the significance of the results along with any limitation and improvements which could be made;

- *Conclusion*: We use this word as a "topic" word to orient the reader towards the fact that we will be summarising the main points of the essay or paper.

Even the table of content of a book can be seen as a type of introduction. Why? How so? Look at the table of content of any textbook in your own discipline. Can you read the table of content as a coherent piece of text which guides the reader as to what is to come over the course of the textbook?